GRAVITATIONAL VACUUM POLARIZATION

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The energy conditions of classical Einstein gravity fail once quantum effects are introduced. These quantum violations of the energy conditions are not subtle high-energy Planck scale effects. Rather the quantum violations of the energy conditions already occur in semiclassical quantum gravity and are first-order $O(\hbar)$ effects. Quantum violations of the energy conditions are widespread, albeit small.

1 Introduction

The energy conditions of classical Einstein gravity are used to prove many powerful and general theorems such as the singularity theorems, the positive mass theorem, and the topological censorship theorem. While quantum mechanical violation of the energy conditions was certainly expected, it is perhaps a little surprising just how widespread the quantum violations of the energy conditions are. Quantum induced violations of the energy conditions are now known to occur in:

- Casimir effect. [1,2]

- Squeezed vacuum. [1,2]

- Cosmological inflation. [2–4]

- Cosmological particle production. [5]

- Conformal anomaly. [6]

- Gravitational vacuum polarization. [7–10]

2 Gravitational vacuum polarization

When a gravitational field acts as an external source applied to a quantum field theory (QFT) it will distort the quantum vacuum, and shift the expectation value of the stress-energy so that $\langle T^{\mu\nu} \rangle \neq 0$. Typically $\langle T^{\mu\nu} \rangle \approx \hbar c / L^4$, where L is some position-dependent characteristic length scale, which may be associated either with the quantum state, or with the background geometry. If the gravitational field of interest contains an event horizon then there are at least four different natural definitions of the quantum mechanical vacuum state:

- Hartle–Hawking vacuum [thermal equilibrium at infinity].

- Boulware vacuum [empty at infinity].

- Unruh vacuum [evaporating black hole].

- Vacuum cleaner vacuum [accreting black hole].

These different states correspond to different definitions of normal ordering on the spacetime. If the spacetime does not possess an event horizon (star, planet) then these particular complications go away and you only have one vacuum state to deal with—the Boulware vacuum. (Corresponding to normal ordering with respect to the usual static t coordinate.)

In contrast to QFT defined on flat Minkowski space, in curved space no analytic calculations are currently possible. One resorts either to numerical methods [11,12] or to nonperturbative analytic approximations. To keep matters tractable, I confine attention to massless conformally coupled scalar QFT on the Schwarzschild geometry. All calculations are performed in the test-field limit, and back reaction is not included. (For recent progress regarding back reaction see Hochberg–Popov–Sushkov. [13])

3 Violations in the Hartle–Hawking vacuum

In the Hartle–Hawking vacuum, the energy condition violations are confined to the region between the event horizon and the unstable photon orbit at $r = 3M$. The dominant energy condition (DEC) is the first energy condition to be violated at $r = 2.992M$, next the weak energy condition (WEC) is violated at $r = 2.438M$, finally the null energy condition (NEC) and strong energy condition (SEC) are violated once $r = 2.298M$. You can show this by using the numerically calculated values of the stress-energy tensor. [7,11] For consistency, you can check that the same qualitative features survive when using Page's analytic approximation. [7] The fact that the DEC violations first occur so close to the unstable photon orbit is suggestive, but may merely be a numerical accident.

4 Violations in the Boulware vacuum

In the Boulware vacuum all the pointwise energy conditions are violated throughout the entire region exterior to the event horizon. [8] This is proved by using the numerically calculated values of the stress-energy tensor. [8,11] Again, for consistency, you can check that the same qualitative features survive when using the Page–Brown–Ottewill analytic approximation. [8] This property should continue to hold for the external region outside a star or planet, the presence of the event horizon not being the crucial feature in deriving this result. [8] Working perturbatively around flat space, Flanagan and Wald [14] have verified that these energy condition violations are generic to first order in GM/r.

5 Violations in the Unruh vacuum

In the Unruh vacuum all the pointwise energy conditions are violated throughout the entire region exterior to the event horizon. [10] Again, this is best shown by using the numerically calculated values of the stress-energy tensor. [10–12] You have to be careful since testing the energy conditions involves subtracting numbers that

are numerically close to each other. At large distances the stress-energy tensor is dominated by the Hawking flux, but the Hawking flux quietly cancels out of the NEC evaluated on outgoing null geodesics, so it is the subdominant pieces of the stress-energy tensor that are critical in testing for energy condition violations. For good measure, no good analytic approximation is known for the Unruh vacuum, and the numerical data are all that one has to work with. [10,12]

6   Violations in $(1+1)$ dimensions

In $(1 + 1)$ dimensions the expectation value of the stress–energy can be calculated analytically. The exact results obtained for this case are qualitatively in agreement with the pattern found (numerically and/or via analytic approximation) in $(3 + 1)$ dimensions. [9] This serves as a useful sanity check on the entire formalism.

7   Discussion

Quantum violations of the energy conditions are widespread, albeit small. There are important issues of principle at stake: Present versions of the positive mass theorem and the singularity theorems are strictly classical and do not survive the introduction of even semiclassical quantum effects. (The hypotheses of these theorems are known to be false in semiclassical quantum gravity.) There are suspicions that it might be possible to place general bounds on the size of the quantum violations [15] and so still be able to prove some type of generalized singularity theorem and/or generalized positive mass theorem, but it is far from clear how to go about this.

Acknowledgments

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